Alternatives to Explicit State Space Search
Decoupled Search

Daniel Gnad & Álvaro Torralba

ICAPS'17 Tutorial

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About us

Daniel Gnad

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About you

Target audience:

Ideally, you are..

- .. familiar with Classical Planning Formalisms (FDR/SAS$^+$).
- .. familiar with Planning as Heuristic Search.
- .. aware of an important issue in Explicit State Space Search
  $\rightarrow$ State Space Explosion

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Don't hesitate to ask questions if something is unclear!

Agenda

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Classical Planning

Definition. *A planning task is a 4-tuple* $\Pi = (V, A, I, G)$ *where:*

- V *is a set of state variables, each* $v \in V$ *with a finite domain* D_v.
- A *is a set of actions; each* $a \in A$ *is a triple* $(pre(a), \mathit{eff}(a), c(a))$, *of precondition and effect (partial assignments), and the action's cost* $c(a) \in \mathbb{R}^{0+}$.
- *Initial state* I *(complete assignment), goal* G *(partial assignment).*

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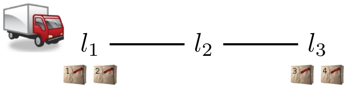

- $V = \{t, p_1, p_2, p_3, p_4\}$ with $D_t = \{l_1, l_2, l_3\}$ and $D_{p_i} = \{t, l_1, l_2, l_3\}$.
- $A = \{load(p_i, x), unload(p_i, x), drive(x, x')\}$

Semantics – The State Space of a Planning Task

Definition. *Let $\Pi = (V, A, I, G)$ be an FDR planning task. The state space of Π is the labeled transition system $\Theta_\Pi = (S, L, c, T, I, S^G)$ where:*

- *The states S are the complete variable assignments.*
- *The labels $L = A$ are Π's actions; the cost function c is that of Π.*
- *The transitions are $T = \{s \xrightarrow{a} s' \mid pre(a) \subseteq s, s' = s[\![a]\!]\}$.*
  If $pre(a) \subseteq s$, then a is applicable in s and, for all $v \in V$, $s[\![a]\!][v] := \mathit{eff}(a)[v]$ if $\mathit{eff}(a)[v]$ is defined and $s[\![a]\!][v] := s[v]$ otherwise.
  If $pre(a) \not\subseteq s$, then $s[\![a]\!]$ is undefined.
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$\rightarrow$ Solution ("Plan"): Action sequence mapping I into $s \in S^G$.
Optimal plan: Minimum summed-up cost.

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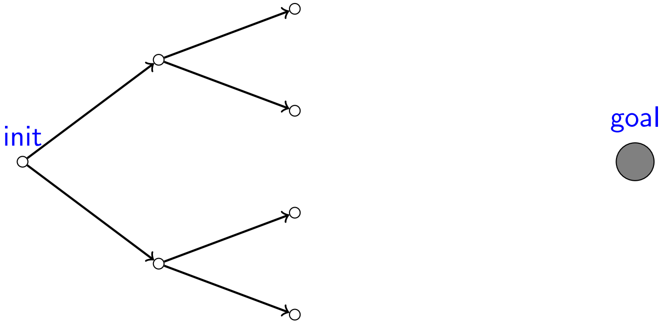
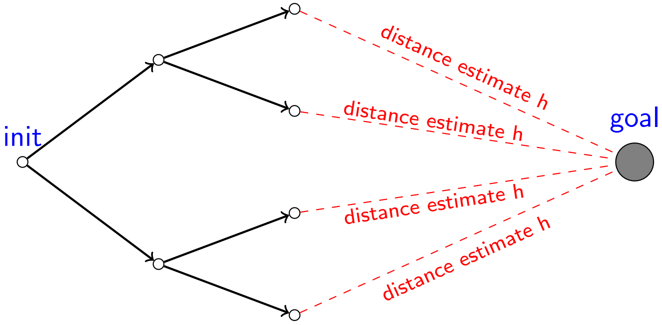


A successful approach: Heuristic Search

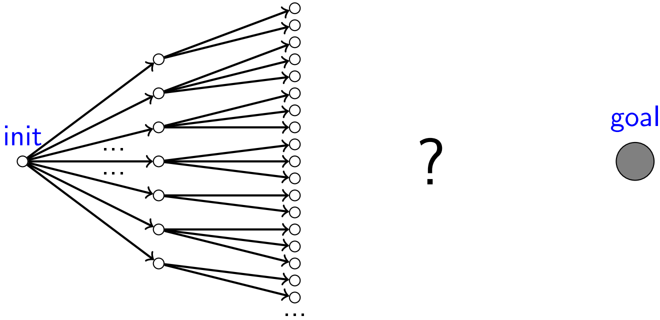


→ Forward state space search. Heuristic function h maps states s to an estimate $h(s)$ of goal distance.

Alternatives to State Space Search (not covered here)

- **Planning as SAT**: Extensions use, e. g., heuristics, symmetry breaking.
  Kautz and Selman (1992, 1996); Ernst *et al.* (1997); Rintanen (1998, 2003, 2012)
- **Property Directed Reachability**
  Bradley (2011); Eén *et al.* (2011); Suda (2014)
- Planning via **Petri Net Unfolding**
  Godefroid and Wolper (1991); McMillan (1992); Esparza *et al.* (2002); Edelkamp *et al.* (2004); Hickmott *et al.* (2007); Bonet *et al.* (2008, 2014)
- **Partial-order Planning**
  Sacerdoti (1975); Kambhampati *et al.* (1995); Younes and Simmons (2003); Bercher *et al.* (2013)
- **Factored Planning** (details later)
  Knoblock (1994); Amir and Engelhardt (2003); Brafman and Domshlak (2006); Kelareva *et al.* (2007); Brafman and Domshlak (2008, 2013); Fabre *et al.* (2010)
- ...

State Space Explosion



Huge branching factor $\rightarrow$ state space *explosion*.

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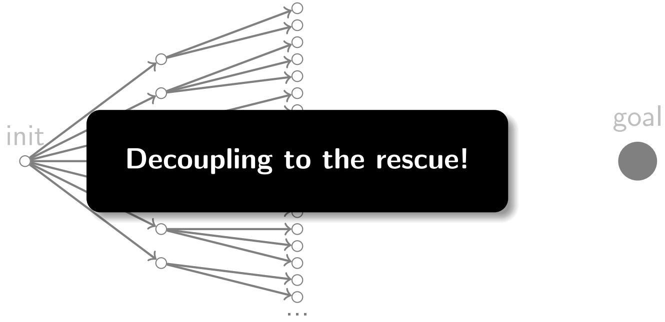

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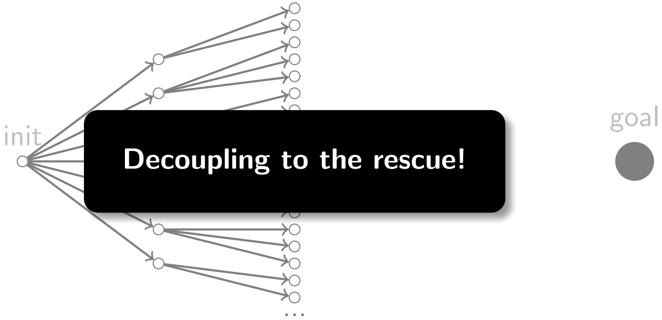

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Helmert and Röger (2008)

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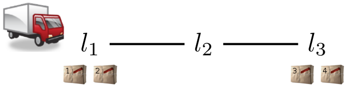
Star-Topology Decoupling



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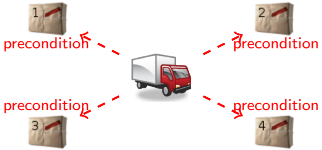

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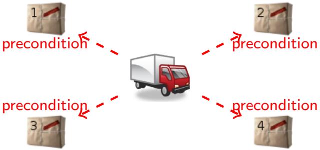

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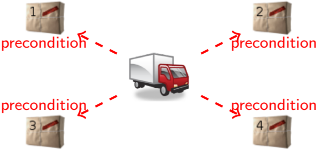

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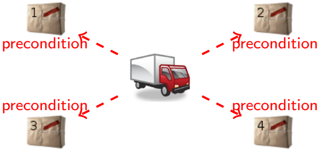

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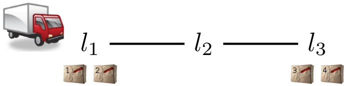

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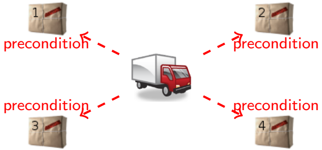

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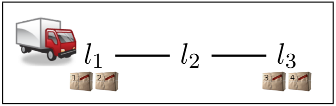

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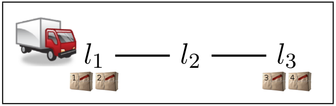

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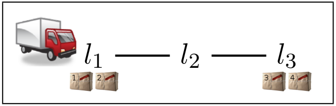

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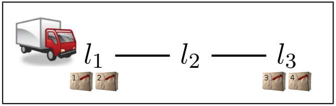

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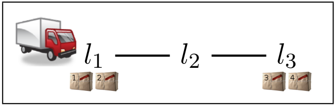

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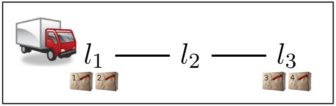

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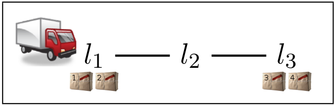

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We can choose (a) or (b) for each of p_1 and p_3 independently

$\Longrightarrow$ **Maintain the compliant paths for each leaf separately.**

Exponential Reduction of the State Space

Domain	Success: Std	Success: POR	Success: Unfold.	Success: Decoupled	Representation Size (in Thousands): Std	Representation Size (in Thousands): POR	Representation Size (in Thousands): Decoupled
Reachable State Space. Right: Average over Instances Commonly Built							
Solvable Benchmarks: From the International Planning Competition (IPC)							
Depots	4	4	2	**5**	30,954.8	30,954.8	**3,970.0**
Driverlog	5	5	3	**10**	35,632.4	35,632.4	**127.2**
Elevators	21	17	3	**41**	22,652.1	22,651.1	**186.7**
Logistics	12	12	11	**27**	3,793.8	3,793.8	**8.2**
Miconic	50	45	30	**145**	52,728.9	52,673.1	**2.4**
NoMystery	11	11	7	**40**	29,459.3	25,581.5	**10.0**
Pathways	**4**	**4**	3	**4**	54,635.5	**1,229.0**	11,211.9
PSR	3	3	3	3	39.4	33.9	**11.1**
Rovers	5	**6**	4	5	98,051.6	6,534.4	**4,032.9**
Satellite	**5**	**5**	**5**	4	2,864.2	582.5	**352.7**
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Woodworking	11	20	**22**	16	438,638.5	**226.8**	9,688.9
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Decoupled Search can be viewed as a form of Petri Net Unfolding, exploiting the star topology to avoid the hardness of detecting the reachable markings.

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- Center States s^C: complete assignment to F^C.
- Leaf States s^L: complete assignment to an $F^L \in \mathcal{F}^L$.

Decoupled States

Definition *Let* $\mathcal{F}$ *be a star factoring with center* F^C *and leaves* $\mathcal{F}^L$*. A decoupled state* $s^\mathcal{F}$ *is a triple* $(\pi^C(s^\mathcal{F}), \text{center}(s^\mathcal{F}), prices(s^\mathcal{F}))$ *where* $\pi^C(s^\mathcal{F})$ *is a center path,* $\text{center}(s^\mathcal{F})$ *is a center state, and* $prices(s^\mathcal{F})$ *is a pricing function,* $prices(s^\mathcal{F}) : S^L \mapsto \mathbb{R}^{0+} \cup \{\infty\}$*, mapping each leaf state to a non-negative price.*

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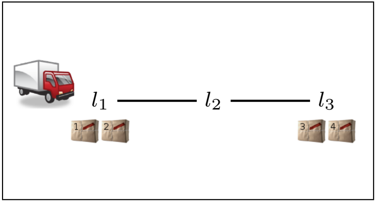

	l_1	t	l_2	l_3
p_1	0	1	∞	∞

	l_1	t	l_2	l_3
p_2	0	1	∞	∞

$t = l_1$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0

The Compliant-Path Graph

Center path:

$l_1 \xrightarrow{\text{drive}(l_1, l_2)} l_2 \xrightarrow{\text{drive}(l_2, l_3)} l_3 \xrightarrow{\text{drive}(l_3, l_2)} l_2 \xrightarrow{\text{drive}(l_2, l_1)} l_1$

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(a) $l_1 \xrightarrow{\text{load}(p_1, l_1)} t \xrightarrow{\text{unload}(p_1, l_2)} l_2$

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Definition *Let $\pi^C = \langle a^C_1, \dots, a^C_n \rangle$ be a center path traversing center states $\langle s^C_0, \dots, s^C_n \rangle$. The π^C-compliant-path graph for a leaf $F^L \in \mathcal{F}^L$, denoted* $\mathrm{CompG}[\pi^C, F^L]$, *is the arc-labeled weighted directed graph whose vertices are $\{s^L_t \mid s^L \in S^L[F^L], 0 \leq t \leq n\}$, and whose arcs are:*

(i) $s^L_t \xrightarrow{a^L} s'^L_t$ *with weight $c(a^L)$ whenever $s^L, s'^L \in S^L[F^L]$ and $a^L \in A^L[F^L] \setminus A^C$ are such that $s^C_t[\mathcal{V}(pre(a^L)) \cap F^C] = pre(a^L)[F^C]$, $s^L[\mathcal{V}(pre(a^L)) \cap F^L] = pre(a^L)[F^L]$, and $s^L[\![a^L]\!] = s'^L$.*

(ii) $s^L_t \xrightarrow{0} s'^L_{t+1}$ *with weight 0 whenever $s^L, s'^L \in S^L[F^L]$ are such that $s^L[\mathcal{V}(pre(a^C_t)) \cap F^L] = pre(a^C_t)[F^L]$ and $s^L[\![a^C_t]\!] = s'^L$.*

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The Compliant-Path Graph – Example

Center path π^C:

l_1

CompG$[\pi^C, \{p_1\}]$:

$(p_1 = t)_0$

1 (un)load(p_1, l_1)

$\underline{(p_1 = l_1)_0}$

The Compliant-Path Graph – Example

Center path π^C:

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CompG$[\pi^C, \{p_1\}]$:

$$\begin{array}{l} (p_1 = t)_0 \\ 1 \quad \text{(un)load}(p_1, l_1) \\ \underline{(p_1 = l_1)_0} \end{array}$$

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$$\begin{tikzcd}
(p_1 = t)_0 \arrow[r, "0"] \arrow[d, leftrightarrow, bend right, "1"', "\text{(un)load}(p_1, l_1)"] & (p_1 = t)_1 \arrow[dd, leftrightarrow, bend left, "1", "\text{(un)load}(p_1, l_2)"'] \\
\underline{(p_1 = l_1)_0} \arrow[r, "0"] & (p_1 = l_1)_1 \\
& (p_1 = l_2)_1
\end{tikzcd}$$

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\end{tikzcd}$$

The Compliant-Path Graph – No-Empty Example

Center path π^C: $pre(drive(l_x, l_y, \mathbf{p_z})) = \{t = l_x, \mathbf{p_z} = \mathbf{t}\}$

l_1

CompG$[\pi^C, \{p_1\}]$:

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(un)load(p_1, l_2) 1

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CompG$[\pi^C, \{p_1\}]$:

$$(p_1 = t)_0 \xrightarrow{0} (p_1 = t)_1$$

$(p_1 = t)_0 \leftrightarrow \underline{(p_1 = l_1)_0}$: 1 (un)load(p_1, l_1)

$(p_1 = t)_1 \leftrightarrow (p_1 = l_2)_1$: (un)load$(p_1, l_2)$ 1

The Compliant-Path Graph – No-Empty Example

Center path π^C: $pre(drive(l_x, l_y, \mathbf{p_z})) = \{t = l_x, \mathbf{p_z = t}\}$

$$l_1 \xrightarrow{\text{drive}(l_1, l_2, p_1)} l_2 \xrightarrow{\text{drive}(l_2, l_3, p_1)} l_3$$

CompG$[\pi^C, \{p_1\}]$:

$$\begin{tikzcd}
(p_1 = t)_0 \arrow[r, "0"] \arrow[d, bend right, "1\ \text{(un)load}(p_1, l_1)"'] & (p_1 = t)_1 \arrow[r, "0"] \arrow[d, bend left, "\text{(un)load}(p_1, l_2)\ 1"] & (p_1 = t)_2 \arrow[d, bend left, "\text{(un)load}(p_1, l_3)\ 1"] \\
\underline{(p_1 = l_1)_0} \arrow[u, bend left] & (p_1 = l_2)_1 \arrow[u, bend right] & (p_1 = l_3)_2 \arrow[u, bend right]
\end{tikzcd}$$

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Center path π^C: $pre(drive(l_x, l_y, \mathbf{p_z})) = \{t = l_x, \mathbf{p_z} = \mathbf{t}\}$

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CompG$[\pi^C, \{p_1\}]$:

$$\begin{tikzcd}
(p_1 = t)_0 \arrow[r, "0"] \arrow[d, leftrightarrow, "1\ \text{(un)load}(p_1, l_1)"'] & (p_1 = t)_1 \arrow[r, "0"] \arrow[d, leftrightarrow, "\text{(un)load}(p_1, l_2)\ 1"] & (p_1 = t)_2 \arrow[dd, leftrightarrow, "\text{(un)load}(p_1, l_3)\ 1"] \\
\underline{(p_1 = l_1)_0} & (p_1 = l_2)_1 & \\
& & (p_1 = l_3)_2
\end{tikzcd}$$

The Compliant-Path Graph – No-Empty Example

Center path π^C: $pre(drive(l_x, l_y, \mathbf{p_z})) = \{t = l_x, \mathbf{p_z} = \mathbf{t}\}$

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CompG$[\pi^C, \{p_1\}]$:

$$(p_1 = t)_0 \xrightarrow{0} (p_1 = t)_1 \xrightarrow{0} (p_1 = t)_2 \xrightarrow{0} (p_1 = t)_3$$

$(p_1 = t)_0 \leftrightarrow (p_1 = l_1)_0$: 1 (un)load$(p_1, l_1)$

$\underline{(p_1 = l_1)_0}$

$(p_1 = t)_1 \leftrightarrow (p_1 = l_2)_1$: (un)load$(p_1, l_2)$ 1

$(p_1 = t)_2 \leftrightarrow (p_1 = l_3)_2$: (un)load$(p_1, l_3)$ 1

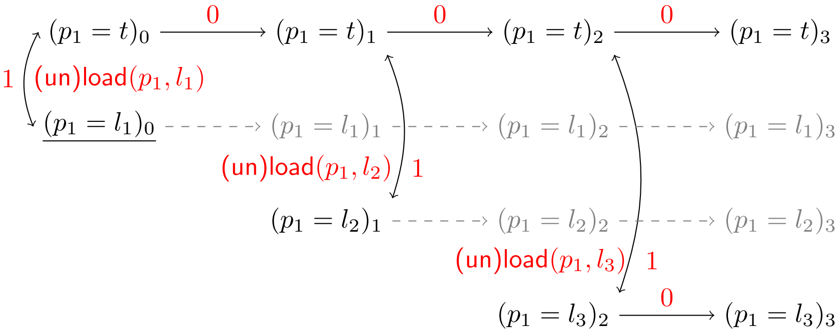
$$(p_1 = l_3)_2 \xrightarrow{0} (p_1 = l_3)_3$$

The Compliant-Path Graph – No-Empty Example

Center path π^C: $pre(drive(l_x, l_y, \mathbf{p_z})) = \{t = l_x, \mathbf{p_z} = \mathbf{t}\}$

$$l_1 \xrightarrow{\text{drive}(l_1, l_2, p_1)} l_2 \xrightarrow{\text{drive}(l_2, l_3, p_1)} l_3 \xrightarrow{\text{drive}(l_3, l_2, p_2)} l_2$$

CompG$[\pi^C, \{p_1\}]$:



Decoupled State Space

Definition *The decoupled state space is a labeled transition system* $\Theta^{\mathcal{F}}_{\Pi} = (S^{\mathcal{F}}, A^C, c|_{A^C}, T^{\mathcal{F}}, I^{\mathcal{F}}, S^{\mathcal{F}}_G)$ *as follows:*

(i) $S^{\mathcal{F}}$ *is the set of all decoupled states.*

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(i) $S^{\mathcal{F}}$ *is the set of all decoupled states.*

(ii) $I^{\mathcal{F}}$ *is the decoupled initial state, where* center$(I^{\mathcal{F}}) := I[F^C]$, $\pi^C(I^{\mathcal{F}}) := \langle\rangle$, *and, for every leaf* $F^L \in \mathcal{F}^L$ *and leaf state* $s^L \in S^L[F^L]$, $prices(I^{\mathcal{F}})[s^L]$ *is the cost of a cheapest path from* $I[F^L]_0$ *to* s^L_0 *in* CompG$[\langle\rangle, F^L]$.

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(iii) $S^{\mathcal{F}}_G$ *are the decoupled goal states* s_G, *where* $G[F^C] \subseteq \mathrm{center}(s_G)$ *and, for every* $F^L \in \mathcal{F}^L$, *there exists a leaf goal state* $s^L \in S^L[F^L]$ *s.t.* $G[F^L] \subseteq s^L$ *and* $prices(s_G)[s^L] < \infty$.

Decoupled State Space – *Cont.*

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Decoupled State Space – *Cont.*

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(ii) $T^{\mathcal{F}}$ *contains a transition* $(s^{\mathcal{F}} \xrightarrow{a^C} t^{\mathcal{F}}) \in T^{\mathcal{F}}$ *whenever* $a^C \in A^C$ *and* $s^{\mathcal{F}}, t^{\mathcal{F}}$ *are such that:*

Decoupled State Space – *Cont.*

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1. $\pi^C(s^{\mathcal{F}}) \circ \langle a^C \rangle = \pi^C(t^{\mathcal{F}})$;

Decoupled State Space – *Cont.*

Definition *The decoupled state space is a labeled transition system* $\Theta_\Pi^\mathcal{F} = (S^\mathcal{F}, A^C, c|_{A^C}, T^\mathcal{F}, I^\mathcal{F}, S_G^\mathcal{F})$ *as follows:*

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1. $\pi^C(s^\mathcal{F}) \circ \langle a^C \rangle = \pi^C(t^\mathcal{F})$;
2. $\mathrm{center}(s^\mathcal{F})[\mathcal{V}(pre(a^C)) \cap F^C] = pre(a^C)[F^C]$;

Decoupled State Space – *Cont.*

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2. $\mathsf{center}(s^\mathcal{F})[\mathcal{V}(pre(a^C)) \cap F^C] = pre(a^C)[F^C]$;
3. $\mathsf{center}(s^\mathcal{F})[\![a^C]\!] = \mathsf{center}(t^\mathcal{F})$;

Decoupled State Space – *Cont.*

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1. $\pi^C(s^{\mathcal{F}}) \circ \langle a^C \rangle = \pi^C(t^{\mathcal{F}})$;
2. $\text{center}(s^{\mathcal{F}})[\mathcal{V}(pre(a^C)) \cap F^C] = pre(a^C)[F^C]$;
3. $\text{center}(s^{\mathcal{F}})[\![a^C]\!] = \text{center}(t^{\mathcal{F}})$;
4. *for every* $F^L \in \mathcal{F}^L$ *where* $\mathcal{V}(pre(a^C)) \cap F^L \neq \emptyset$, *there exists* $s^L \in S^L[F^L]$ *s.t.* $s^L[\mathcal{V}(pre(a^C)) \cap F^L] = pre(a^C)[F^L]$ *and* $prices(s)[s^L] < \infty$; *and*

Decoupled State Space – *Cont.*

Definition *The decoupled state space is a labeled transition system* $\Theta^{\mathcal{F}}_{\Pi} = (S^{\mathcal{F}}, A^C, c|_{A^C}, T^{\mathcal{F}}, I^{\mathcal{F}}, S^{\mathcal{F}}_G)$ *as follows:*

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1. $\pi^C(s^{\mathcal{F}}) \circ \langle a^C \rangle = \pi^C(t^{\mathcal{F}})$;
2. center$(s^{\mathcal{F}})[\mathcal{V}(pre(a^C)) \cap F^C] = pre(a^C)[F^C]$;
3. center$(s^{\mathcal{F}})[\![a^C]\!] =$ center$(t^{\mathcal{F}})$;
4. *for every* $F^L \in \mathcal{F}^L$ *where* $\mathcal{V}(pre(a^C)) \cap F^L \neq \emptyset$*, there exists* $s^L \in S^L[F^L]$ *s.t.* $s^L[\mathcal{V}(pre(a^C)) \cap F^L] = pre(a^C)[F^L]$ *and* $prices(s)[s^L] < \infty$*; and*
5. *for every leaf* $F^L \in \mathcal{F}^L$ *and leaf state* $s^L \in S^L[F^L]$*,* $prices(t^{\mathcal{F}})[s^L]$ *is the cost of a cheapest path from* $I[F^L]_0$ *to* s^L_n *in* CompG$[\pi^C(t^{\mathcal{F}}), F^L]$*, where* $n := |\pi^C(t^{\mathcal{F}})|$*.*

Decoupled Search – Example

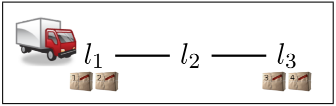
	l_1	t	l_2	l_3
p_1	0	1	∞	∞

	l_1	t	l_2	l_3
p_2	0	1	∞	∞

$t = l_1$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0



Decoupled Search – Example

	l_1	t	l_2	l_3
p_1	0	1	∞	∞

	l_1	t	l_2	l_3
p_2	0	1	∞	∞

$t = l_1$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0

$\xrightarrow{\text{drive}(l_1, l_2)}$

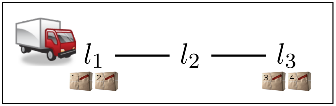
	l_1	t	l_2	l_3
p_1	0	1	**2**	∞

	l_1	t	l_2	l_3
p_2	0	1	**2**	∞

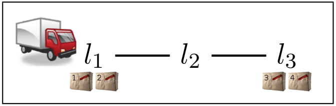
$t = l_2$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0



Decoupled Search – Example



	l_1	t	l_2	l_3
p_1	0	1	∞	∞

	l_1	t	l_2	l_3
p_2	0	1	∞	∞

$t = l_1$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0

drive(l_1, l_2)

	l_1	t	l_2	l_3
p_1	0	1	**2**	∞

	l_1	t	l_2	l_3
p_2	0	1	**2**	∞

$t = l_2$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0

drive(l_2, l_3)

	l_1	t	l_2	l_3
p_1	0	1	2	**2**

	l_1	t	l_2	l_3
p_2	0	1	2	**2**

$t = l_3$

	l_1	t	l_2	l_3
p_3	∞	**1**	∞	0

	l_1	t	l_2	l_3
p_4	∞	**1**	∞	0

Decoupled Search – No-Empty Example

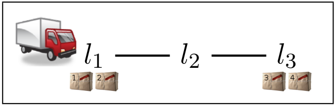
	l_1	t	l_2	l_3
p_1	0	1	∞	∞

	l_1	t	l_2	l_3
p_2	0	1	∞	∞

$t = l_1$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0



Decoupled Search – No-Empty Example

	l_1	t	l_2	l_3
p_1	0	1	∞	∞

	l_1	t	l_2	l_3
p_2	0	1	∞	∞

$t = l_1$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0

$\xrightarrow{\text{drive}(l_1, l_2, p_1)}$

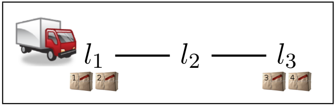
	l_1	t	l_2	l_3
p_1	∞	1	**2**	∞

	l_1	t	l_2	l_3
p_2	0	1	**2**	∞

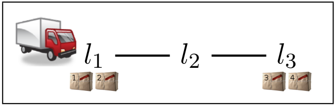
$t = l_2$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0



Decoupled Search – No-Empty Example



	l_1	t	l_2	l_3
p_1	0	1	∞	∞

	l_1	t	l_2	l_3
p_2	0	1	∞	∞

$t = l_1$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0

drive(l_1, l_2, p_1)

	l_1	t	l_2	l_3
p_1	∞	1	**2**	∞

	l_1	t	l_2	l_3
p_2	0	1	**2**	∞

$t = l_2$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0

drive(l_2, l_3, p_2)

	l_1	t	l_2	l_3
p_1	∞	1	2	**2**

	l_1	t	l_2	l_3
p_2	∞	1	∞	**2**

$t = l_3$

	l_1	t	l_2	l_3
p_3	∞	**1**	∞	0

	l_1	t	l_2	l_3
p_4	∞	**1**	∞	0

Hypercubes

Definition *A state* p *in* Π *is a member state of a decoupled state* $s^{\mathcal{F}}$*, if* $p[F^C] = \text{center}(s^{\mathcal{F}})$ *and, for all leaves* $F^L \in \mathcal{F}^L$*,* $prices(s^{\mathcal{F}})[p[F^L]] < \infty$*. We say that* p *has cost* $cost_{s^{\mathcal{F}}}(p)$ *in* $s^{\mathcal{F}}$*, where* $cost_{s^{\mathcal{F}}}(p) := \sum_{F^L \in \mathcal{F}^L} prices(s^{\mathcal{F}})[p[F^L]]$*. The hypercube of* $s^{\mathcal{F}}$*, denoted* $[s^{\mathcal{F}}]$*, is the set of all member states of* $s^{\mathcal{F}}$*.*

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	l_1	t	l_2	l_3
p_1	∞	1	2	2

	l_1	t	l_2	l_3
p_2	∞	1	∞	2

$t = l_3$

	l_1	t	l_2	l_3
p_3	∞	1	∞	0

	l_1	t	l_2	l_3
p_4	∞	1	∞	0

Decoupled State $s^{\mathcal{F}}$

$tl_3, p_1t, p_2l_3, p_3l_3, p_4l_3$ 3

$tl_3, p_1l_3, p_2l_3, p_3t, p_4t$ 6

$tl_3, p_1t, p_2t, p_3t, p_4l_3$ 3

$tl_3, p_1l_3, p_2t, p_3t, p_4l_3$ 4

$tl_3, p_1t, p_2l_3, p_3t, p_4t$ 5

...

Hypercube $[s^{\mathcal{F}}]$ (24 member states!)

Hypercubes

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	l_1	t	l_2	l_3
p_1	∞	1	2	2

	l_1	t	l_2	l_3
p_2	∞	1	∞	2

$t = l_3$

	l_1	t	l_2	l_3
p_3	∞	1	∞	0

	l_1	t	l_2	l_3
p_4	∞	1	∞	0

Decoupled State $s^{\mathcal{F}}$

$tl_3, p_1t, p_2l_3, p_3l_3, p_4l_3$ 3

$tl_3, p_1l_3, p_2l_3, p_3t, p_4t$ 6

$tl_3, p_1t, p_2t, p_3t, p_4l_3$ 3

$tl_3, p_1l_3, p_2t, p_3t, p_4l_3$ 4

$tl_3, p_1t, p_2l_3, p_3t, p_4t$ 5

...

Hypercube $[s^{\mathcal{F}}]$ (24 member states!)

Hypercube dimensions = Leaves; Axis values = Leaf States.

Hypercubes II

$[s^{\mathcal{F}}]$ **contains all states reachable via a path** π **that contains** $\pi^{C}(s^{\mathcal{F}})$**.**

Hypercubes II

$[s^{\mathcal{F}}]$ **contains all states reachable via a path** π **that contains** $\pi^{C}(s^{\mathcal{F}})$**.**

Center path:

$$l_1 \xrightarrow{\text{drive}(l_1, l_2)} l_2 \xrightarrow{\text{drive}(l_2, l_3)} l_3 \xrightarrow{\text{drive}(l_3, l_2)} l_2 \xrightarrow{\text{drive}(l_2, l_1)} l_1$$

:

$$l_1 \xrightarrow{\text{load}(p_1, l_1)} t \xrightarrow{\text{unload}(p_1, l_2)} l_2$$

$$\ldots$$

$$l_1 \xrightarrow{\text{load}(p_1, l_1)} t \qquad \xrightarrow{\text{unload}(p_1, l_3)} l_3$$

:

$$l_3 \xrightarrow{\text{load}(p_3, l_3)} t \xrightarrow{\text{unload}(p_3, l_2)} l_2$$

$$\ldots$$

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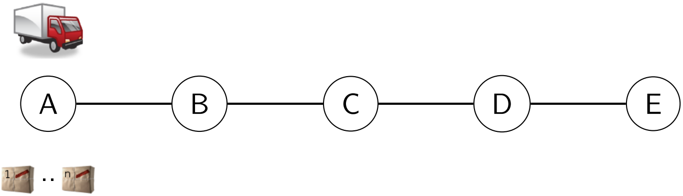
State Space Size Reduction

Illustrative Example:



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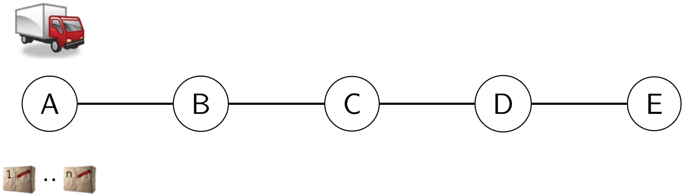

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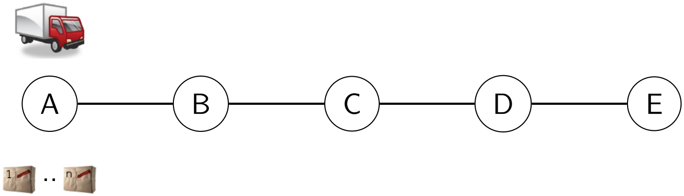
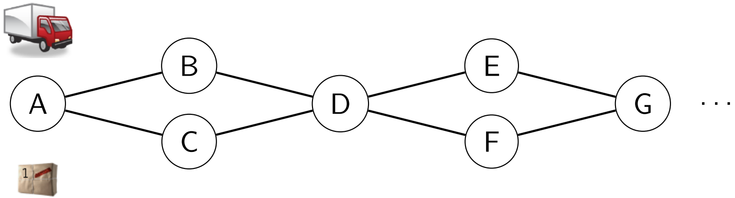


- E.g. loading the packages at A:
  2^n **reachable standard states but only a single decoupled state!**
- Standard state space size here is $5 * 6^n$; decoupled is 15.

Exponential Blow-Up

Illustrative Example:



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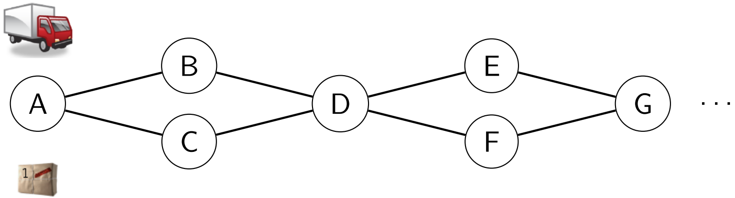

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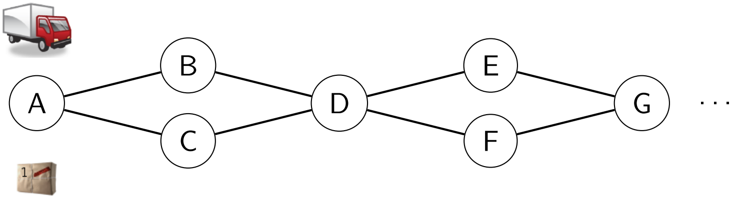

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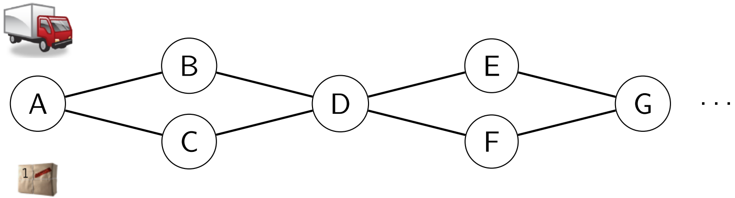

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- Multiple packages: both effects occur.
  Length of map vs. # of packages.

Agenda

Planning Heuristics

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Definition *For a state $s \in S$, the perfect heuristic value h^* of s is the cost of an optimal plan for s, or ∞ if there exists no plan for s.*

$\rightarrow$ Heuristic functions h estimate the remaining cost h^*.

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Center heuristics h^C estimate h^{C*}, and star heuristics h^S estimate h^{S*}.

$\rightarrow$ But how to compute such heuristics?

Heuristic Compilation

Connect to standard classical planning heuristics!

- **Center heuristics:** Set leaf action costs to 0,

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A^L_{aux} changes for every decoupled state $s^{\mathcal{F}}$!

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p_1	0	1	∞	2

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$t = l_3$

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- Heuristic admissible? $\rightarrow$ We can guarantee optimality!

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Goal price $= 2 + 0 + 1 = 3$
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Any standard (complete/optimal) search on the modified graph yields a (complete/optimal) decoupled search algorithm.

$$\mathbf{A}^* \rightarrow \mathbf{DA}^*$$

Hypercubes

$tl_3, p_1t, p_2l_3, p_3l_3, p_4l_3$ $h = 5$

$tl_3, p_1l_3, p_2l_3, p_3t, p_4t$ $h = 7$

$tl_3, p_1t, p_2t, p_3t, p_4l_3$ $h = 4$

$tl_3, p_1l_3, p_2t, p_3t, p_4l_3$ $h = 2$

$tl_3, p_1t, p_2l_3, p_3t, p_4t$ $h = 5$

$\dots$

Estimate the minimum heuristic value of all member states.

$\rightarrow$ Captured by A^L_{aux}.

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Agenda

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$\rightarrow$ **Guarantees finiteness of decoupled state space!**

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Previously seen state:

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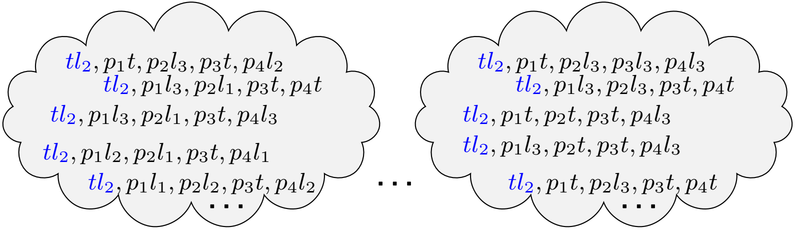
Definition *A decoupled state $s^{\mathcal{F}}$ dominates another state $t^{\mathcal{F}}$, if the center state is the same and* **for all leaf states** s^L*:* $prices(s^{\mathcal{F}})[s^L] \leq prices(t^{\mathcal{F}})[s^L]$.

Hypercube Pruning



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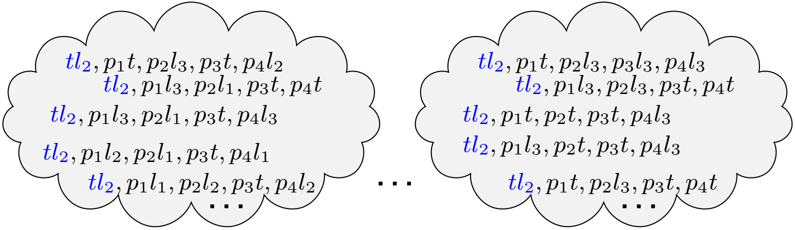

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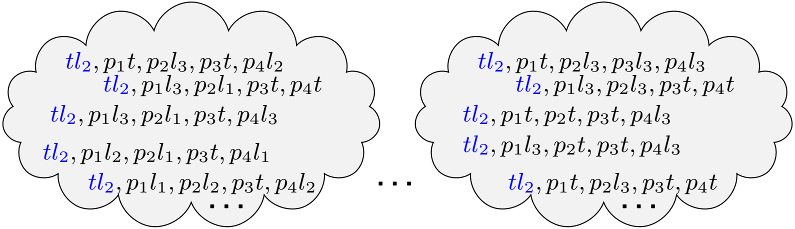

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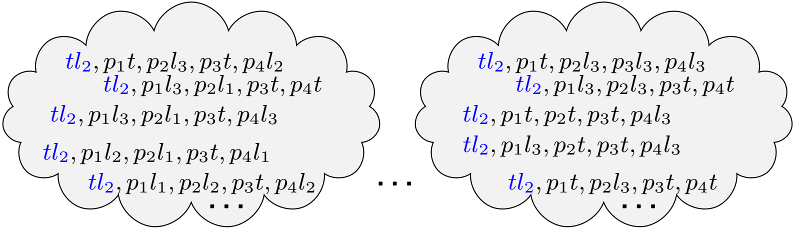

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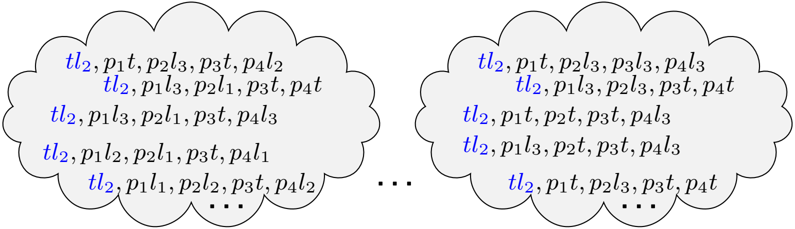

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Extension to optimality? $\rightarrow$ Need to take prices into account.

Agenda

Variable Dependencies

Definition *The causal graph of* Π *is the directed graph* $CG(\Pi)$ *with vertices* V *and an arc* $u \rightarrow v$ *if* $u \neq v$ *and there exists an action* $a \in A$ *so that either (i) there exists* $a \in A$ *so that* $pre(a)[u]$ *and* $\mathit{eff}(a)[v]$ *are both defined, or (ii) there exists* $a \in A$ *so that* $\mathit{eff}(a)[u]$ *and* $\mathit{eff}(a)[v]$ *are both defined.*

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Causal graphs capture variable dependencies.

Definition *The interaction graph of Π given $\mathcal{F}$ is the directed graph $\mathsf{IG}_{\Pi}(\mathcal{F})$, with vertices $\mathcal{F}$, and an arc $F \rightarrow F'$ if $F \neq F'$, and there exist $v \in F$ and $v' \in F'$, s.t. $v \rightarrow v'$ is an arc in $CG(\Pi)$.*

The interaction graph is the quotient of $CG(\Pi)$ over $\mathcal{F}$.

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- Design factors motivated by abstraction hierarchies or agents.
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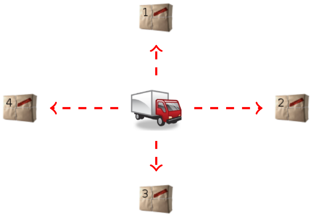
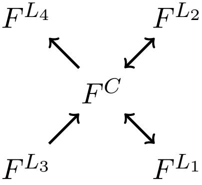
Star Factoring: Force the factoring to induce a star profile!



$\rightarrow$ That is, choose factoring $\mathcal{F}$ so that the interaction graph has a center incident to all arcs.

Factoring Types

Recap:
$\mathcal{F}$ is a star factoring if $|\mathcal{F}| > 1$ and there exists $F^C \in \mathcal{F}$ such that, for every action a where $\mathcal{V}(\mathit{eff}(a)) \cap F^C = \emptyset$, there exists $F^L \in \mathcal{F}^L$ with $\mathcal{V}(\mathit{eff}(a)) \subseteq F$ and $\mathcal{V}(\mathit{pre}(a)) \subseteq F \cup F^C$.



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$$\begin{tikzcd}
F^{L_4} & & F^{L_2} \arrow[dd, leftrightarrow] \\
& F^C \arrow[ul] \arrow[ur, leftrightarrow] \arrow[dr, leftrightarrow] & \\
F^{L_3} \arrow[ur] \arrow[rr] & & F^{L_1}
\end{tikzcd}$$

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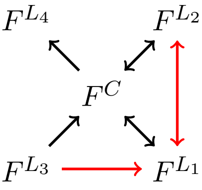


Strict-Star Factoring:
Arbitrary center-leaf interaction; no dependencies between leaves directly.

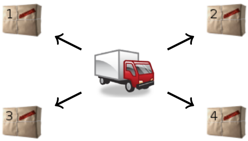
Definition *A factoring $\mathcal{F}$ is a strict-star factoring, if there exists a center factor $F^C \in \mathcal{F}$ such that the arcs in $\mathsf{IG}_\Pi(\mathcal{F})$ are in* $\{F^C \rightarrow F^L \mid F^L \in \mathcal{F} \setminus \{F^C\}\} \cup \{F^C \leftarrow F^L \mid F^L \in \mathcal{F} \setminus \{F^C\}\}$.

How to automatically find a factoring?

Based on the SCCs of the causal graph:

Fork / Inverted-Fork Factorings:
Set leaves to causal graph leaf / root components.

Xshape Factoring:
First do fork factoring, then inverted-fork on resulting center.
Need to take care of leaf-leaf dependencies!



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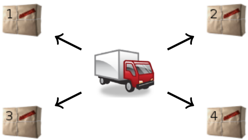
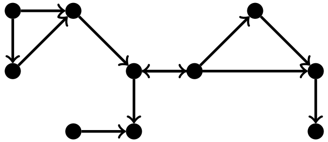


Important criteria:

- **Number of leaves factors** → exponential gain.
- **Leaf flexibility** → *frozen* leaves.
- Leaf size (domain size product) → runtime/memory overhead.
- Structure of leaf state space.

Maximizing the Number of Leaves

Maximum number of leaves in a strict-star factoring?
$\rightarrow$ size of a **maximum independent set (MIS)** of the causal graph.



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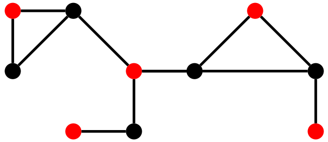

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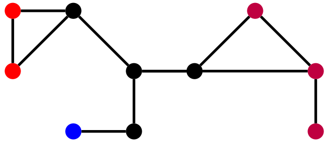

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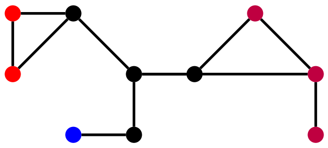
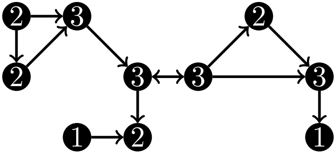
3. **Abstain** if less than 2 leaves.



A Greedy Approach – # Incident Arcs (IA)

What if computing an MIS is infeasible?



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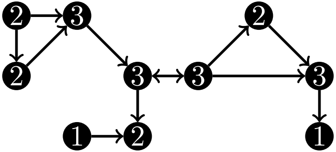

IA Strategy:

1. Move variables most densely connected in causal graph to center.

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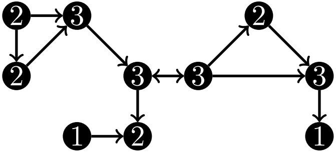

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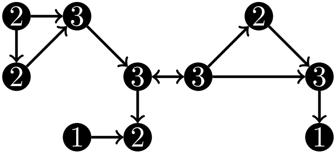

IA Strategy:

1. Move variables most densely connected in causal graph to center.
2. Leaves = weakly connected components in $CG[V \setminus F^C]$.
3. Select factoring with maximum number of **mobile** leaves.
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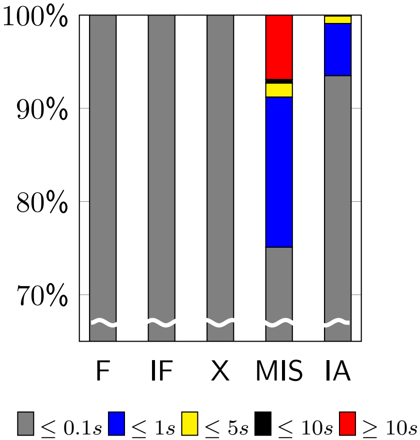
Factoring Time



Agenda

The interesting stuff . . . after the break

Short Break (5 min)

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Implementation in Fast Downward

The interesting stuff . . . after the break

Short Break (5 min)

Implementation in Fast Downward

Hands-On – Implement your own heuristic in decoupled search

Instructions:

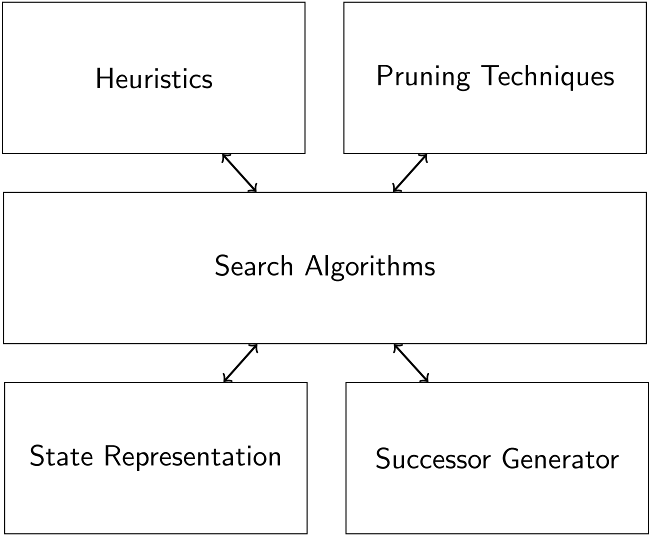
hg clone `https://bitbucket.org/dagnad/decoupled-fd`

hg up icaps-tutorial

The relevant files are `../src/search/icaps_heuristic.*`

The URL is linked on `http://fai.uni-saarland.de/software.html` ("Decoupled Fast Downward").

Decoupled Search within Fast Downward (Helmert (2006))



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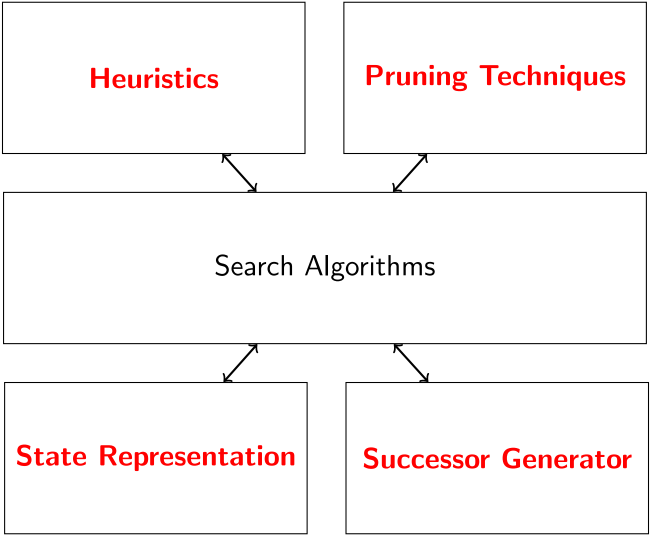

Representing Decoupled States

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Decoupled State: a pair of

- Center State + **auxiliary variable** to distinguish decoupled states with the same center,
- Instance of "CompliantPathGraph" (many sub-classes).

$\rightarrow$ Search Algorithms use (augmented) center state only.

$\rightarrow$ Accessing pricing function via a vector indexed by aug. center state ID.

Representing Decoupled States II

CompliantPathGraphs:
Many different variants for **optimal/satisficing** planning, more elaborate **dominance pruning** methods, **symbolic leaves**, ...

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- A `vector<int>` for each leaf factor.
- vector indexed by leaf state IDs.

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Many different variants for **optimal/satisficing** planning, more elaborate **dominance pruning** methods, **symbolic leaves**, ...

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Update of Pricing Function:

- Run uniform-cost search from currently reached leaf states.
- Cache entire leaf state spaces to be more efficient.

Interface to Heuristics/Pruning/Successor Generator

```
CPGStorage::get_cpg(State)
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→ Global access to the pricing function of a decoupled state.

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Access price/reachability of a leaf state by its ID + factor.

Functions:

- `has_leaf_state(id, factor)`,
- `get_cost_of_state(id, factor)`,
- `get_number_state(factor)`,
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Wanna Take a Look at the Code?

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It's a ~~trap~~ mess!

Hands-On Decoupled Search in Fast Downward

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Want to implement a simple heuristic function?

A mixture between goal-counting and PDB heuristic!

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`g_min_goal_cost`: precomputed minimum goal cost with "patterns" $\mathcal{F}^L$.

Build: `./build_all`

Execute: `./fast-downward.py [task-file] --decoupling "fork" --search "astar(icaps)"`

Agenda

What's Done

Basic Framework is Established! Gnad and Hoffmann (2015); Gnad *et al.* (2015)

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+ lots of extensions:

- **Factoring methods**, target-profile factoring. Gnad *et al.* (2017a)
  → Tomorrow at HSDIP!
- **Partial-order Reduction**. Gnad *et al.* (2016)
- **Symmetry Breaking**. Gnad *et al.* (2017c)
  → Friday morning!
- **Dominance Pruning**. Torralba *et al.* (2016)
- Combination with **symbolic search**, BDDs to represent leaf state spaces. Gnad *et al.* (2017b)

Alternative Planning Formulations

It works for Classical Planning, but what about . . .

- **Numerical Planning?**
- **Probabilistic Planning?**
- **Temporal Planning?**
- **Generalized Planning?**

Beyond Delete-Relaxation

What we have:

$$h^{\mathsf{max}}, h^{\mathsf{add}}, h^{\mathsf{FF}}, h^{\mathsf{LM\text{-}cut}}$$

How to connect to **other types of heuristics**?

- **Abstraction Heuristics**
  - Patter Database Heuristics (PDB)
  - Merge-And-Shrink
  - Cartesian Abstractions
- **Linear-Programming-based Heuristics**
  - Operator Counting
  - Potential Heuristics
- **Landmark Heuristics**
- ...

Beyond Classical Planning

Why scramble for star topologies in IPC benchmarks when the world is full of applications that have a star topology by definition?

- **Multiple agents** (leaves) that interact on a set of shared variables (center).
- **Model Checking!** E.g., client-server architectures; parallel processors with central memory (weak memory models).
- Decoupled Search vs. **Petri Net Unfolding**.

Take Home Message

Explicit Search is the most prominent approach to tackle classical planning problems.

- Full generality (A^*, GBFS, hill-climbing, ...).
- Powerful heuristics.
- Various search enhancements available (e. g., successor pruning)
- Works well across all (IPC) domains.

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- More specialized way to exploit, e.g., conditional independence.
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Alternative State Representations like Decoupled Search offer:

- More specialized way to exploit, e.g., conditional independence.
- Potentially **exponential gain** over explicit search.
- Easy to Use:
    - Implemented in Fast Downward.
    - Many heuristics + pruning methods applicable.
    - Factoring process is fast!

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