Alternatives to Explicit State Space Search
Symbolic Search

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June 19, 2017

About us

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About you

Target audience:

Ideally, you are..

- .. familiar with Classical Planning Formalisms (FDR/SAS$^+$).
- .. familiar with Planning as Heuristic Search.
- .. aware of an important issue in Explicit State Space Search
  $\rightarrow$ State Space Explosion

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Don't hesitate to ask questions if something is unclear!

About the tutorial

Symbolic Search:

There have been many tutorials on the usefulness of Decision Diagrams:
$\rightarrow$Here: focus on symbolic search algorithms

Agenda

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Classical Planning

Definition. *A planning task is a 4-tuple* $\Pi = (V, A, I, G)$ *where:*

- V *is a set of state variables, each* $v \in V$ *with a finite domain* D_v*.*
- A *is a set of actions; each* $a \in A$ *is a triple* (pre_a, eff_a, c_a)*, of precondition and effect (partial assignments), and the action's cost* $c_a \in \mathbb{R}^{0+}$*.*
- *Initial state* I *(complete assignment), goal* G *(partial assignment).*

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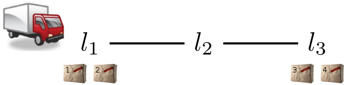
Running Example:



- $V = \{t, p_1, p_2, p_3, p_4\}$
  with $D_t = \{l_1, l_2, l_3\}$ and $D_{p_i} = \{t, l_1, l_2, l_3\}$.
- $A = \{load(p_i, x), unload(p_i, x), drive(x, x')\}$

Semantics – The State Space of a Planning Task

Definition. *Let $\Pi = (V, A, I, G)$ be an FDR planning task. The state space of Π is the labeled transition system $\Theta_\Pi = (S, L, c, T, I, S^G)$ where:*

- *The states S are the complete variable assignments.*
- *The labels $L = A$ are Π's actions; the cost function c is that of Π.*
- *The transitions are $T = \{s \xrightarrow{a} s' \mid pre_a \subseteq s, s' = s[\![a]\!]\}$.*

  If $pre_a \subseteq s$, then a is applicable in s and, for all $v \in V$, $s[\![a]\!][v] := eff_a[v]$ if $eff_a[v]$ is defined and $s[\![a]\!][v] := s[v]$ otherwise.

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→ Solution ("Plan"): Action sequence mapping I into $s \in S^G$.
Optimal plan: Minimum summed-up cost.

A successful approach: Heuristic Search

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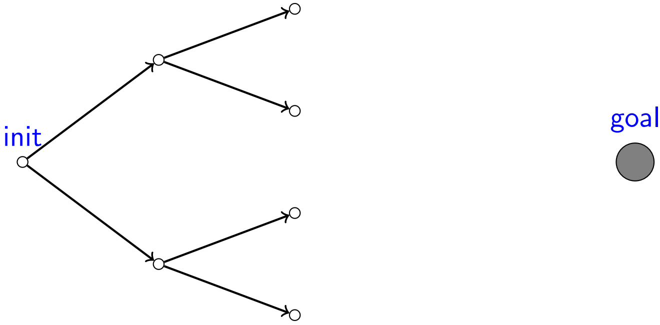
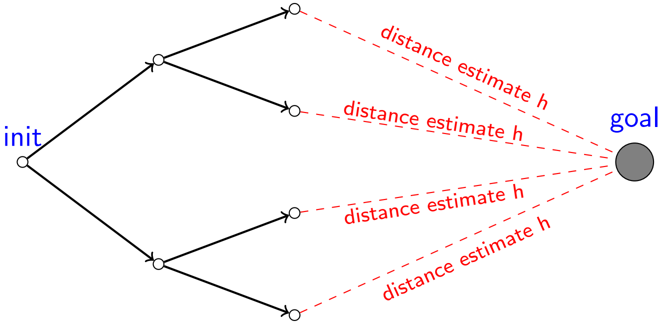


A successful approach: Heuristic Search

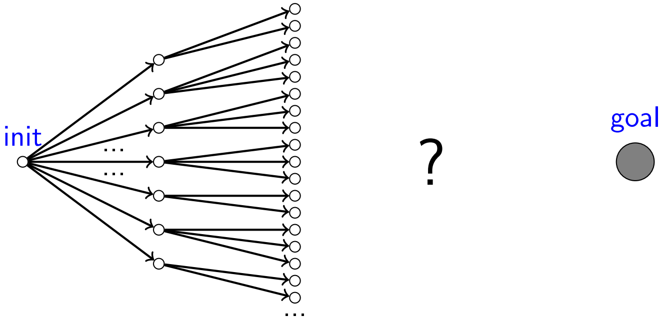


→ Forward state space search. Heuristic function h maps states s to an estimate $h(s)$ of goal distance.

Alternatives to State Space Search (not covered here)

- **Planning as SAT**: Extensions use, e. g., heuristics, symmetry breaking. [KS92, KS96, EMW97, Rin98, Rin03, Rin12]
- **Property Directed Reachability** [Bra11, EMB11, Sud14]
- Planning via **Petri Net Unfolding** [GW91, McM92, ERV02, ELL04, HRTW07, BHHT08, BHK$^+$14]
- **Partial-order Planning** [Sac75, KKY95, YS03, BGB13]
- **Factored Planning** [Kno94, AE03, BD06, KBHT07, BD08, BD13, FJHT10]
- ...

State Space Explosion



Huge branching factor → state space *explosion*

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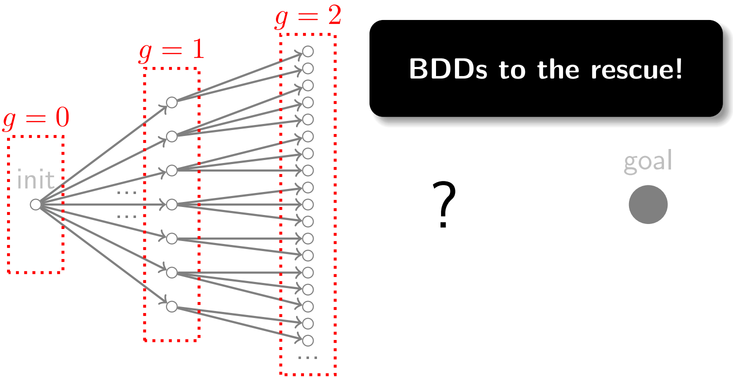

Huge branching factor $\rightarrow$ state space *explosion*

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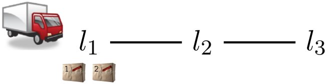
Sets of States as Logical Formulas



$\langle t, l_1 \rangle \land \langle p_1, l_1 \rangle \land \langle p_2, l_1 \rangle$

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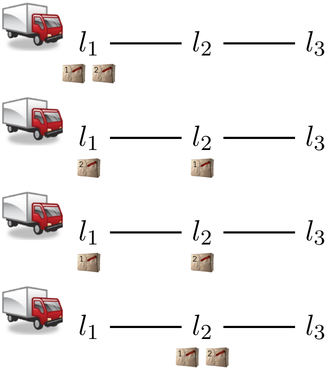

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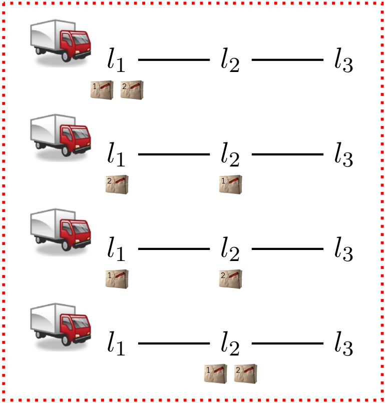

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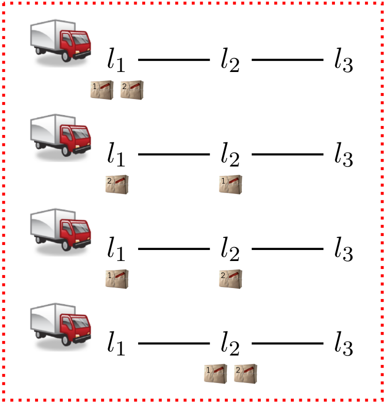

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$$\langle \mathbf{t}, \mathbf{l_1}\rangle \wedge (\langle \mathbf{p_1}, \mathbf{l_1}\rangle \vee \langle \mathbf{p_1}, \mathbf{l_2}\rangle) \wedge (\langle \mathbf{p_2}, \mathbf{l_1}\rangle \vee \langle \mathbf{p_2}, \mathbf{l_2}\rangle)$$

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Operating with Sets of States as Logical Formulas

Sets	Logic
Empty set	$\bot$
All states	$\top$
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Union ($\cup$)	Disjunction ($\vee$)
Inersection ($\cap$)	Conjunction ($\wedge$)
Complement	Negation ($\neg$)

How to Represent Logical Formulas in Practice?

Normal Form/Decision Diagram			
Negation NF (NNF)			
Disjunctive NF (DNF)			
Conjunctive NF (CNF)			
Binary DD (BDD)	[Bry86]		
Zero-sup DD (ZDD)	[Min93]		
Sentential DD (SDD)	[Dar11]		
Determ. DNNF (d-DNNF)	[Dar02]		
Decomp. NNF (DNNF)	[Dar01]		

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Disjunctive NF (DNF)		P	E	E	co-NP	P	co-NP
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Binary DD (BDD)	[Bry86]	E/P	E/P	P	P	P	P
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$*$: In SDDs, $\vee$ and $\wedge$ with compression is not polynomial.

P: polynomial in the size of the representation

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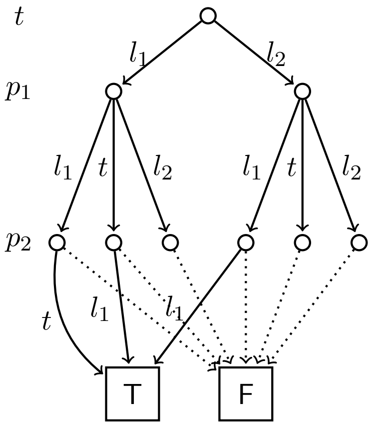
Binary Decision Diagrams (BDDs)

Multi-valued Decision Diagram (MDD)

DAG with a fixed variable ordering.

t	p_1	p_2
l_1	l_1	t
l_2	l_1	l_1
l_1	t	l_1



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Reduction rules:

1. Each node represented only once
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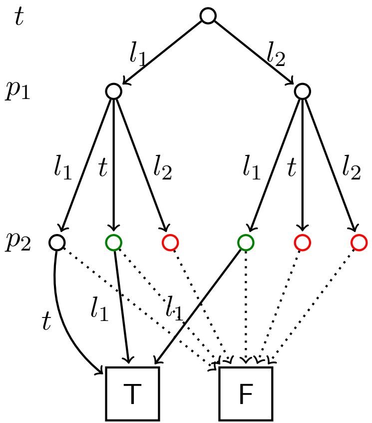

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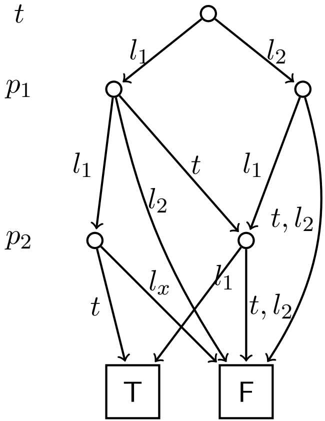

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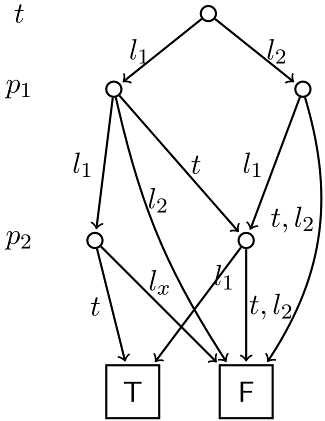

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$\rightarrow$Compilation that uses $log_2|D_v|$ binary variables per FDR variable v

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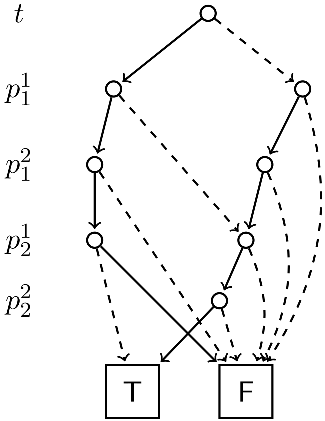

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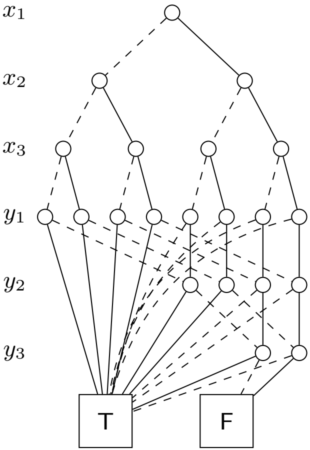
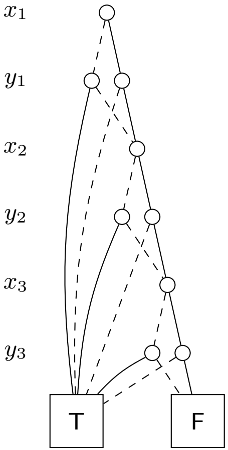
BDD Variable Ordering

$$(x_1 \neq y_1) \vee (x_2 \neq y_2) \vee (x_3 \neq y_3)$$

Exponential ($> 2^{n+1}$)



Polynomial ($3n + 2$)



Practical Strategies for a Good Variable Ordering

Static Variable Ordering: Put causally-related variables close [KE11]

$$\text{Choose ordering } o \text{ that minimizes } \sum_{v_i, v_j \in CG} d_o(v_i, v_j)^2$$

$\rightarrow$No strong theoretical guarantees [KH13] but compares well against other alternatives [BRKM91, CHP93, Mai09, MWBSV88, MIY90]

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Dynamic Variable Ordering: variable re-ordering to minimize the size of the BDDs generated so far

- Finding the optimal BDD ordering is NP-hard [Bry86]
- But practical approximations (based on local-search) exist [Rud93].
- Applied in planning with good results by dynamic-Gamer [KH14]

Complexity Results

BDD Representation of Interesting Sets of States [EK11]

Goal States/Reachable states	Polynomial	Exponential
Polynomial	Gripper	Blocksworld N-puzzle
Exponential	Connect-4 Tic-tac-toe Gomoku	

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Can variable orderings schemas based on the causal graph give us theoretical guarantees for the size of BDDs in the search? [KH14]
$\rightarrow$Mostly not.

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Transition Relation: represents an action a as the relation (set of pairs of states) containing (s, s') where a is applicable in s resulting in s'.

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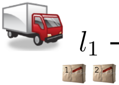
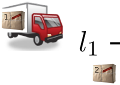


$l_1 - l_2 - l_3$



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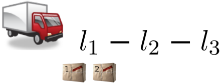

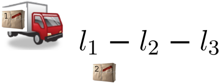

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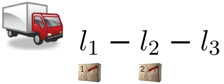

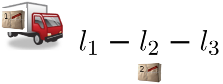

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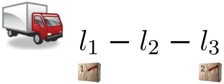

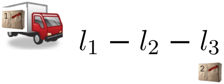

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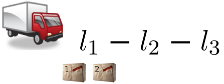

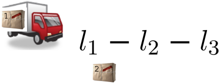

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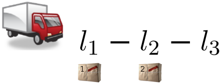

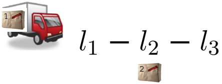

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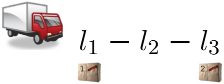

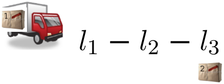

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$\langle p_1, l_1\rangle \wedge \langle t, l_1\rangle \wedge \langle p_1, t\rangle' \wedge \langle t, l_1\rangle' \wedge (\langle p_2, l_1\rangle \leftrightarrow \langle p_2, l_1\rangle') \wedge (\langle p_2, l_2\rangle \leftrightarrow \langle p_2, l_2\rangle') \ldots$

Computing the Successors (Image Computation)

Image: Given a set of states $S(x)$ and a TR $T(x, x')$ generate the successor states

$$\text{image}(S(x), T(x, x')) = \exists x \, . \, S(x) \land T(x, x')[x' \leftrightarrow x]$$

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l_1	l_1	l_1
l_1	l_1	l_2
l_2	l_1	l_3
l_1	l_3	l_1

$T(x, x')$:
$(load(p_1, l_1))$

t	p_1	p_2	t'	p'_1	p'_2
l_1	l_1	l_1	l_1	t	l_1
l_1	l_1	l_2	l_1	t	l_2
l_1	l_1	l_3	l_1	t	l_3

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l_1	l_1	l_3	l_1	t	l_3

Result:

t	p_1	p_2	t'	p'_1	p'_2
l_1	l_1	l_1	l_1	t	l_1
l_1	l_1	l_2	l_1	t	l_2

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l_1	l_1	l_1	l_1	t	l_1
l_1	l_1	l_2	l_1	t	l_2
l_1	l_1	l_3	l_1	t	l_3

Result:

t	p_1	p_2	t'	p'_1	p'_2
l_1	t	l_1			
l_1	t	l_2			

$\exists$-quantification: exponential in the number of variables

Computing the Predecessors (Pre-Image Computation)

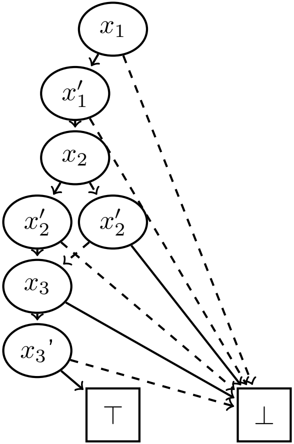
Image: Given a set of states $S(x)$ and a TR $T(x,x')$ generate the predecessor states

$$\text{pre-image}(S(x), T(x,x')) = \exists x' \, . \, S(x)[x' \leftrightarrow x] \wedge T(x,x')$$

- Corresponds to regression [Rin08]

Efficient Image Computation: Variable Ordering

Variable Ordering: Interleave variables x and x'



$\rightarrow$The TR of an action has linear size on the number of variables

Efficient Image Computation

Transition Relation Partitioning [BCL91, JVB08]

Given a set of K actions with the same cost, replace $T_i(x, x')$ and $T_j(x, x')$ by $T_i(x, x') \vee T_j(x, x')$

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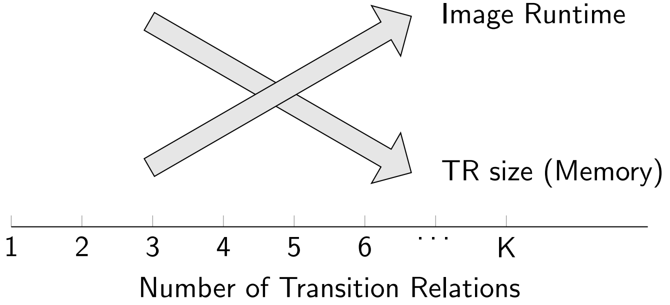

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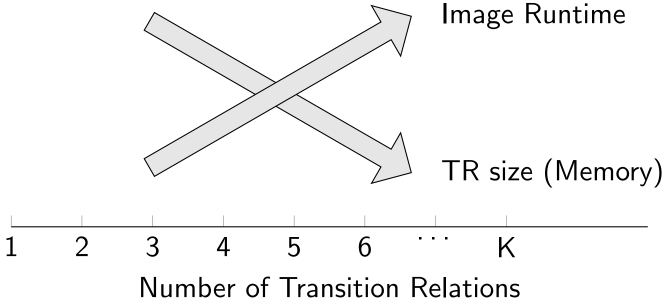

→Merge as many TRs as possible given the available memory [TEK13, TAKE17]

Uses of Decision Diagrams in Classical Planning

- Representation of state-dependent action costs [GKM15]
- Subsumption of partial states [AFB14]
- Dominance pruning [TH15]

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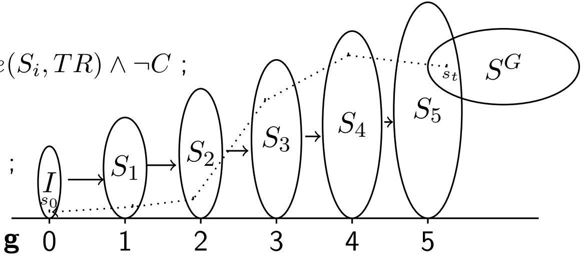
→Here: symbolic search

Agenda

Symbolic Breadth-First Search

Input: Planning Task $\Pi = (V, A, I, G)$
$S_0 \leftarrow I$;
$C \leftarrow \emptyset$;
$i \leftarrow 0$;
while $S_i \neq \emptyset$ **do**
  if $S_i \wedge G$ **then**
    return Plan ;
  end
  $C \leftarrow C \vee S_i$;
  $S_{i+1} \leftarrow image(S_i, TR) \wedge \neg C$;
  $i \leftarrow i + 1$;
end
return Unsolvable ;



Symbolic Uniform-Cost Search

Expand set of states S_i with minimum g-value i

- Zero-cost breadth-first search to obtain all states reachable with $g = i$
- For each TR with action cost c:
  - Use image to compute states reachable with $i + c$
  - Insert the result in the corresponding bucket (disjunction)

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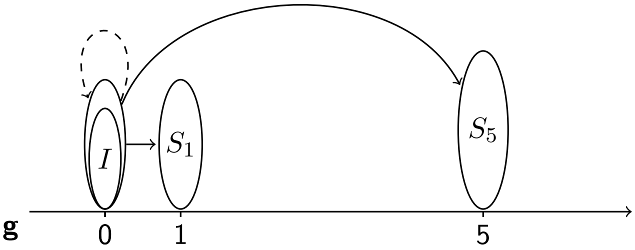

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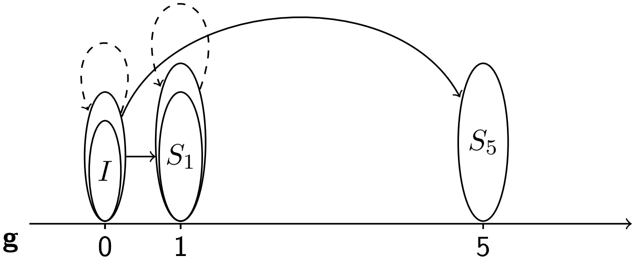

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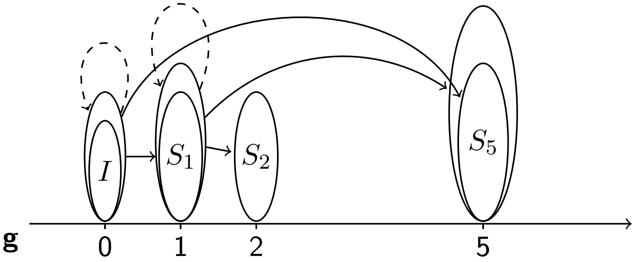

Symbolic Backward Uniform-Cost Search

We can perform the search in backward direction:

- Start with the set of goal states
- Use pre-image instead of image operation

Challenges:

1. Multiple goal states
2. Subsumption of partial states
3. Spurious states

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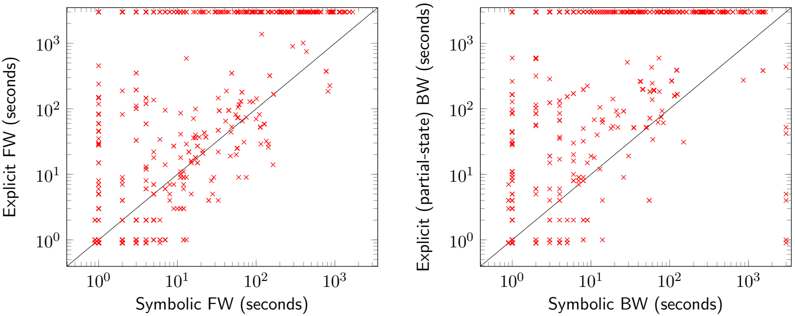
Challenges:

1. Multiple goal states →Not a problem in symbolic search!
2. Subsumption of partial states →Not a problem in symbolic search!
3. Spurious states →Solution: state-invariant pruning [TAKE17]
    - Compute state invariants, e.g., h^2 mutexes
    - Encode the set of spurious states as a BDD
    - Remove spurious states from the goal and the TRs

Symbolic Uniform-Cost Search: Results



Forward Backward

Symbolic Bidirectional Uniform-Cost Search

- Do forward and backward search at the same time
- Decide forward or backward direction at each step

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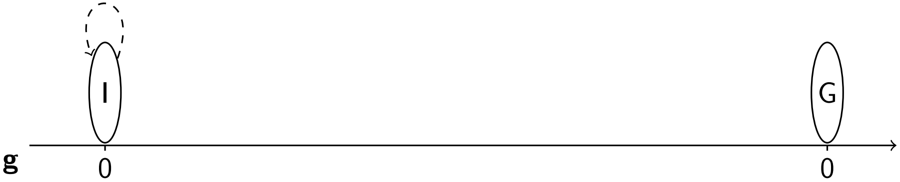
$g_f = 0$ $g_b = 0$



Symbolic Bidirectional Uniform-Cost Search

- Do forward and backward search at the same time
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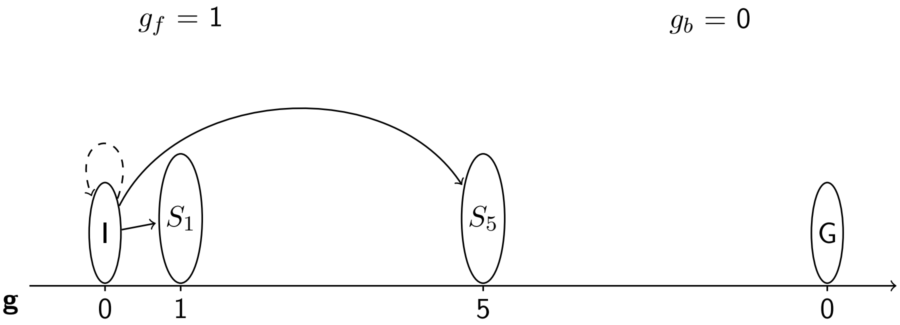
- Stop when $g_f + g_b + min_{a \in A} c(a) \geq Sol$



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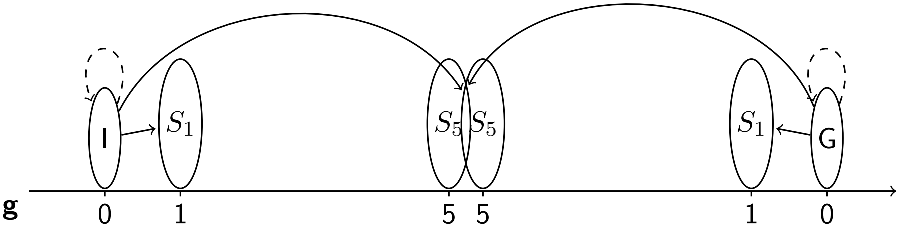
$g_f = 1$ Sol $= 10$ $g_b = 1$



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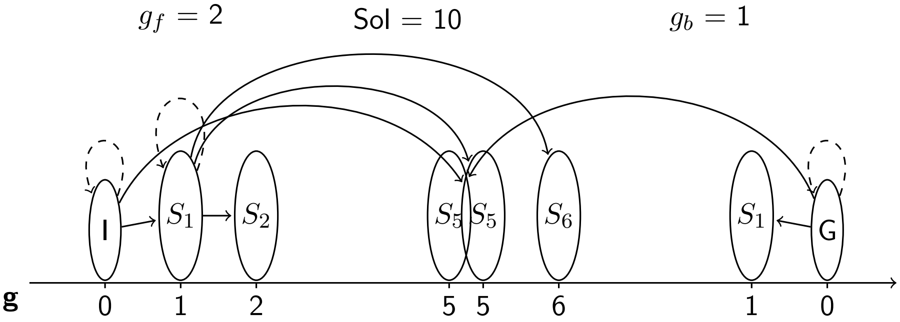

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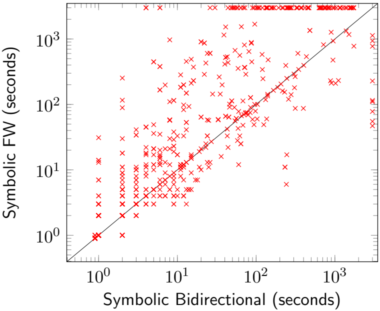
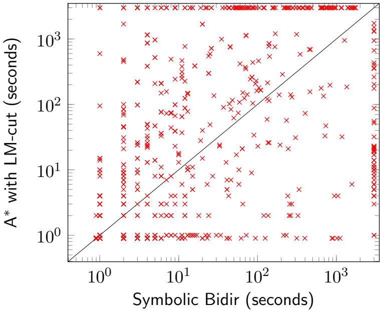


vs Symbolic Forward



vs A* with LM-cut

Agenda

Planning Heuristics

Definition *A* *heuristic* h *is a function* $h : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$*. Its value* $h(s)$ *for a state* s *is referred to as the state's* *heuristic value**, or* h *value.*

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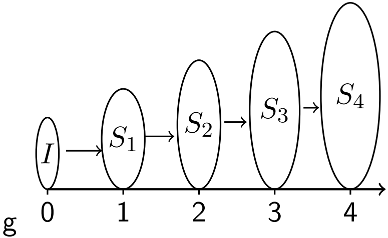
Definition *For a state* $s \in S$*, the perfect heuristic value* h^* *of* s *is the cost of an optimal plan for* s*, or* ∞ *if there exists no plan for* s*.*

$\rightarrow$ Heuristic functions h estimate the remaining cost h^*.

How to Exploit Heuristics in Symbolic Search?



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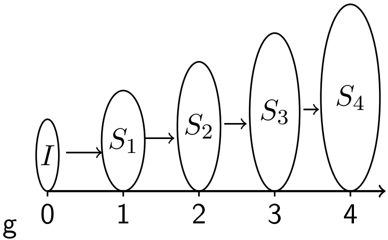

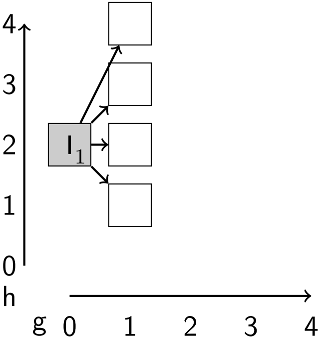

Split a BDD into subsets of states according to their h-value!

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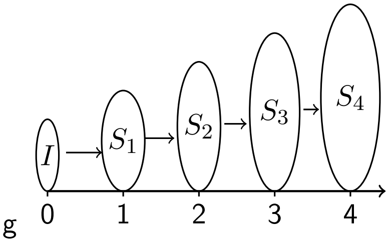

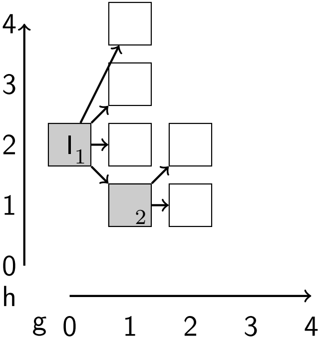

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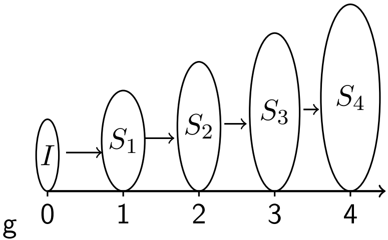

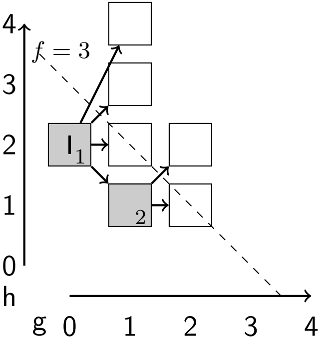

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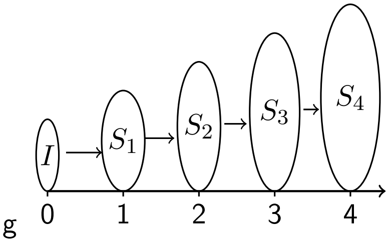

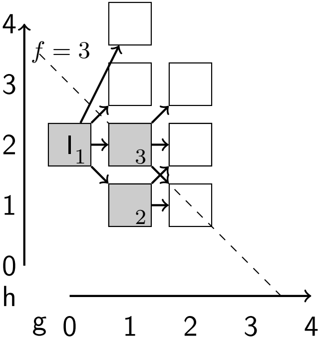

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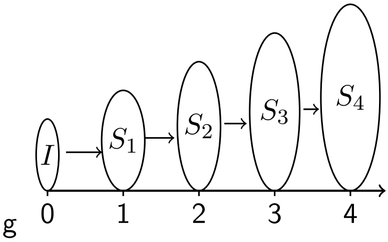

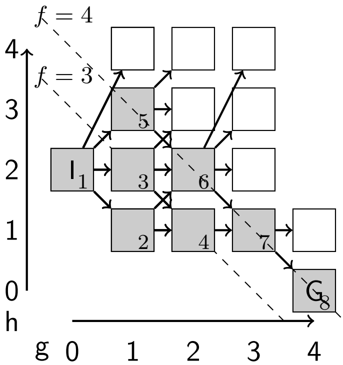

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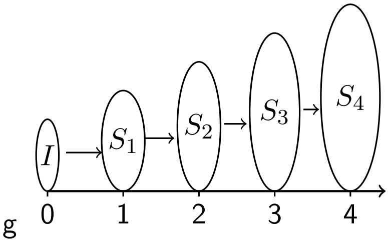

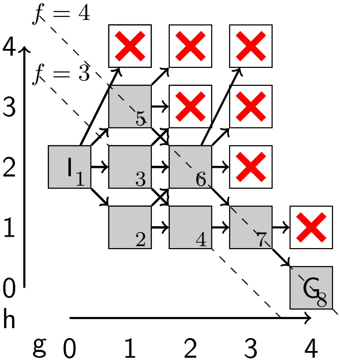

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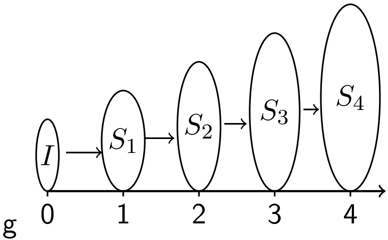

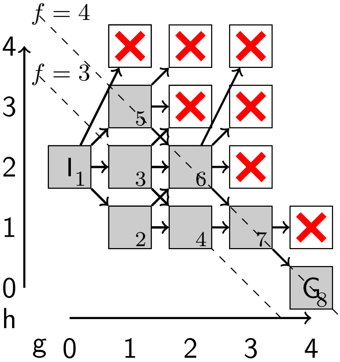

Split a BDD into subsets of states according to their h-value!

- Heuristic computation: how to evaluate a set of states?
- Does the heuristic improve the search performance?

Heuristic Computation: How to Evaluate a Set of States?

1. Iterate over all states in the BDD, computing $h(s)$

Heuristic Computation: How to Evaluate a Set of States?

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2. Precompute the heuristic in form of BDDs: A BDD H_i for each possible h-value representing the set of states with $h(s) = i$

$$\boxed{H_0} \quad \boxed{H_1} \quad \boxed{H_2} \quad \boxed{H_3} \quad \boxed{H_4}$$

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Given a set of states S, split it according to their h-value

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$$\boxed{S}$$

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2. Precompute the heuristic in form of BDDs: A BDD H_i for each possible h-value representing the set of states with $h(s) = i$

Given a set of states S, split it according to their h-value: $S_i = S \wedge B_i$

$$\begin{array}{cccccc} & \boxed{H_0} & \boxed{H_1} & \boxed{H_2} & \boxed{H_3} & \boxed{H_4} \\ \boxed{S} & \wedge & \wedge & \wedge & \wedge & \wedge \\ & \boxed{S_0} & \boxed{S_1} & \boxed{S_2} & \boxed{S_3} & \boxed{S_4} \end{array}$$

Heuristic Computation: How to Evaluate a Set of States?

1. ~~Iterate over all states in the BDD, computing $h(s)$~~
2. Precompute the heuristic in form of BDDs: A BDD H_i for each possible h-value representing the set of states with $h(s) = i$

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$\rightarrow$Can we efficiently precompute a heuristic into BDDs?

- Yes, for some types of abstraction heuristics
- Not in the general case. Finding tractable cases is an open research question!

Agenda

Abstractions

Abstraction: *function* $\alpha : S \mapsto S^\alpha$*.* Induces an abstract state space s.t.:

(i) $I^\alpha = \alpha(I)$.

(ii) $S^{\alpha G} = \{\alpha(s) \mid s \in S^G\}$. /* preserve goal states */

(iii) $T^\alpha = \{(\alpha(s), l, \alpha(t)) \mid (s, l, t) \in T\}$./* preserve transitions */

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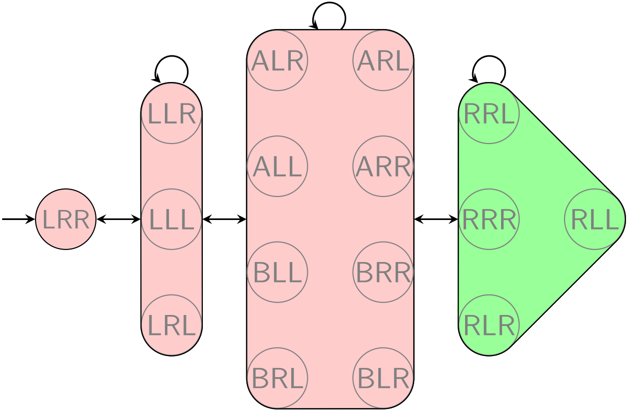
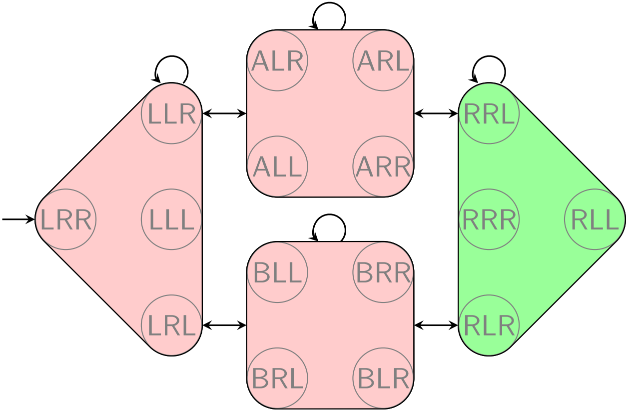


Pattern Databases [CS98, Ede01, HBH+07]

Pattern Databases: Select a subset of variables $V^\alpha \subseteq V$ (pattern). The mapping α is defined as the projection onto V^α.



Abstraction Heuristics

Use the optimal goal-distance in the abstract state space as an (admissible) estimate for the distance in the concrete state space:

$$h(s) = h^*(\alpha(s))$$

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2. Store them in a look-up table
   - $\rightarrow$ In the form of BDDs, so we can use them in BDDA*

Symbolic Pattern Databases

Do symbolic backward uniform-cost search with only a subset of variables

Symbolic Pattern Databases

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- Do not have a limit on the number of variables to consider
- Truncate the search if it takes too much time or memory [AHS07]
- Using all variables: Symbolic Perimeter

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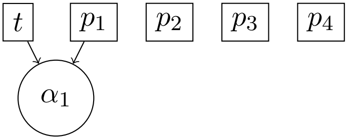
- Using all variables: Symbolic Perimeter

→Really strong for heuristic search planners too [FTLB17]!

Abstraction Heuristics: Merge-and-Shrink [HHHN14, SWH14]

$\boxed{t}$ $\boxed{p_1}$ $\boxed{p_2}$ $\boxed{p_3}$ $\boxed{p_4}$

Abstraction Heuristics: Merge-and-Shrink [HHHN14, SWH14]



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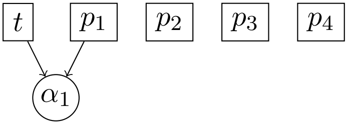

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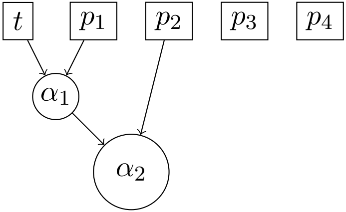
- Shrink strategy: What abstraction to apply to reduce the abstract state space size?
- Merge strategy: What two abstractions to merge next?



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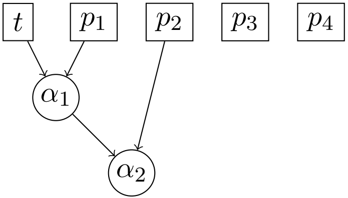

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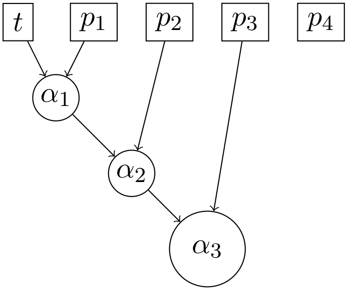

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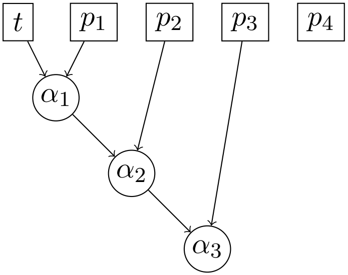

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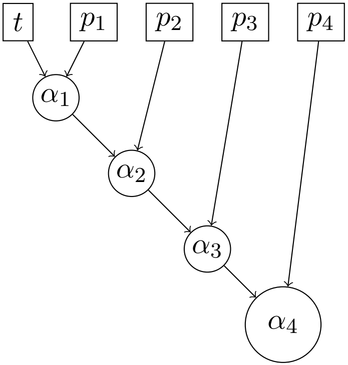

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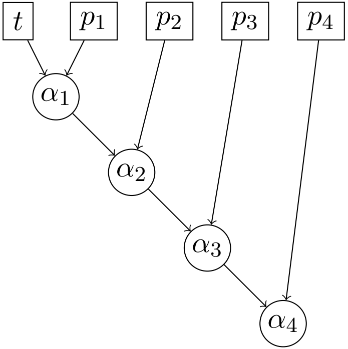

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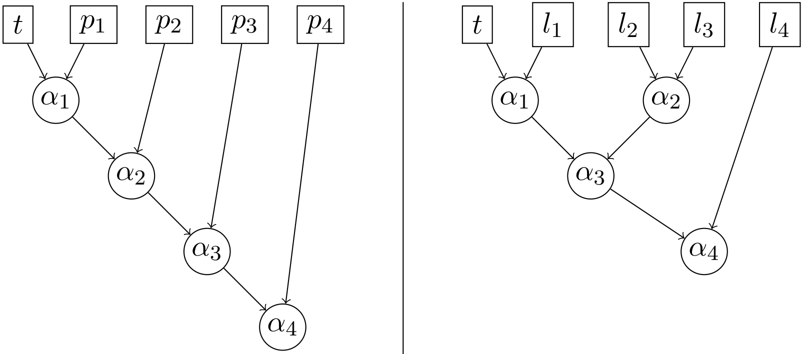

Linear Merge

Non-linear merge

Symbolic Merge-and-Shrink

Are $M\&S$ heuristics efficiently representable by BDDs?

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$\rightarrow$We can use M&S heuristics in symbolic search under some restrictions

SymBA*

- We have good symbolic abstraction heuristics
- But, bidirectional blind search is often the best symbolic search algorihm

Can we use heuristics in symbolic bidirectional search?

SymBA*

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Can we use heuristics in symbolic bidirectional search? Yes! [TGDH16]

1. Start symbolic bidirectional uniform-cost search
   - If it succeeds → done!
2. Detect when it is going to fail and activate heuristics
   - Perimeters abstractions take advantage of the search already performed in the concrete state space [TLB13]

Agenda

1. About this Tutorial
2. Classical Planning: Models, Approaches
3. Symbolic Representation of Planning Tasks
4. Symbolic Blind Search
5. Heuristic Search
6. Symbolic Abstraction Heuristics
7. Implementation
8. Conclusions and Open Challenges

Planners using Symbolic Search

- **MIPS**: Stefan Edelkamp and Malte Helmert
  `http://www.tzi.de/~edelkamp/mips/mips-bdd.html`
- **MIPS-XXL**: Stefan Edelkamp, Shahid Jabbar, and Mohammed Nazih. Extension to net-benefit with external planning
  `http://sjabbar.com/mips-xxl-planner`
- **BDDPlan**: Hans-Peter Strr `http://www.stoerr.net/bddplan.html`
- **Gamer** (IPC08-IPC11): Peter Kissmann and Stefan Edelkamp
  `https://fai.cs.uni-saarland.de/kissmann/planning/downloads/`
  Extensions (IPC14):
  1. cGamer: improved image computation and state invariant pruning
  2. dynamic Gamer: dynamic variable reordering
- **SymBA*** (IPC14): Based on Fast Downward
  `http://fai.cs.uni-saarland.de/torralba/software.html`

BDD Packages

Library	Language	Reference
CUDD	C/C++	[Som]
CacBDD	C++	[LSX13]
BuDDy	C	[CWWG]
CAL	C	
Sylvan	C	[vDvdP15]
JDD	Java	[Vah]
BeeDeeDee	Java	[LMS14]

→Not clear best performer. CUDD, BuDDy, CacBDD have good results in symbolic model-checking [vDHJ+15].
→There are interfaces that adapt these libraries for other languages like Java, Python, Haskell, . . .

Managing BDDs in CUDD in C++

- Supports:
    - BDDs
    - ZDDs
    - ADDs
- Really easy to perform operations:
    - Disjunction (A + B)
    - Conjunction (A * B).
- Possible to set time and memory limits for BDD operations
    $\rightarrow$ Critical to avoid failure due to an exponential-time BDD operation
- Integrated many other potentially useful algorithms
    - BDD minimization
    - Variable re-ordering
    - ...

Symbolic Search in Fast Downward

- SymVariables: BDD representation of FD variables
  - Obtain the BDD that represents a (partial-)state
  - Check if a BDD contains a given state
- SymStateSpaceManager: Search-related structures to a state space
  - Retrieve initial state/goal states
  - Transition relation and image computation
  - State invariants
- Search algorithms: breadth-first, uniform-cost, A^*, bidirectional, ...

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Conclusions

- Symbolic search is useful for classical planning

→ Takes advantage of the structure of the planning task implicitly

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→ Takes advantage of the structure of the planning task implicitly

- Advantages:
  - Compact representation of sets of states
  - Time/memory efficient exploration of state spaces
- Disadvantages:
  - Heuristics/pruning methods must be adapted to leverage the symbolic representation

Explicit vs. Symbolic Search in Cost-Optimal Planning

Algorithms

- A^*
- Uniform-Cost
- Forward
- Bidirectional

Heuristics

- Delete-relaxation: h^{max}, h^+
- Landmarks: h^{LA}, LM-cut
- Abstractions: PDBs, M&S, CEGAR
- Critical paths: h^m
- Flow
- Potentials

max | Disjoint sum

Cost-partitioning

Pruning techniques

- State invariants
- Symmetries
- Partial-order pruning
- Dominance pruning

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Pruning techniques

- State invariants
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And What can YOU do?

Still lots of open questions:

- Are there any alternatives to BDDs?
- Can BDDs be split for a more efficient exploration of the state space?
- Keep optimizing operations
  - How to do efficient image operations [TEK13, TAKE17]?
  - How to choose good BDD variable orderings [KE11, KH13]?

BDDs can be useful for you to represent and operate with sets of states even if you are not planning to use symbolic search!

- Partial-state regression search: Keep set of all states expanded so far [AFB14]
- Dominance Pruning: Keep set of all states dominated by any expanded state [TH15]

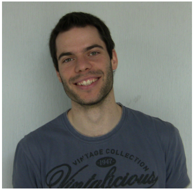
Acknowledgements

Stefan Edelkamp

Peter Kissmann

Jussi Rintanen

Vidal Alcázar



Carlos Linares López

Daniel Borrajo

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